Modelling Modal Age by Various Smoothing Methods with Regard to the Asymmetry in Mortality: the Case of the Czech Republic 1950–2014

Miskolczi Martina, Langhamrova Jitka, Langhamrova Jana
University of Economics in Prague
Department of Demography[1]
Czech Republic

Abstract. The objective of the article is to introduce and compare several methods of smoothing mortality curve, estimating mode and describe statistically age distribution of deaths in the population, separately for males and females. Among considered methods there are statistical models such as Gompertz–Makeham model, logistic models (Himes-Preston-Condran model, Heligman-Pollard model, Thatcher model, Kannisto model), cubic model, polynomial model, Coale-Kisker model, Denuit-Goderniaux model, P-spline-smoothing method based on penalised Poisson likelihood or models with term representing asymmetry. Second, the trend in the period 1950–2014 is analyzed for the case of the Czech Republic, how parameters of parametric distributions change over a time and whether this approach can be used for well-founded and justified projection to the future. Finally, the focus is concentrated on the asymmetry of the age distribution of deaths. Based on the estimated age distribution moments can be expressed, besides the first moment (expected value) also the second moment (variability), the third moment (skewness) and the fourth moment (kurtosis). This is the foundation for the estimate of maximal age that is reached by an individual in the population and, especially, how this maximum is evolving over a time and for each gender, i.e. separately for males and for females. Estimate of age distribution of deaths and its characteristics leads to the assessment of measurement of longevity, estimation of modal age at death for males, females and its convergence and in time, estimation of maximal age, comparison of various methods of mortality smoothing, modal age estimates among each other and comparison of these with other measures, such as life expectancy, median age and life expectancy in health.

Key words: longevity, lifespan, modal age, normal length of life, age distribution of deaths, life expectancy, median age, life tables

[1] Acknowledgement: This article was supported by the Grant Agency of the Czech Republic No. GA ČR 15-13283S under the title *Projection of the Czech Republic Population According to Educational Level and Marital Status* and was processed with contribution of long term institutional support of research activities by Faculty of Informatics and Statistics, University of Economics in Prague.

Extended Abstract

Analysing lifespan and longevity, one of the interesting subject is analysis of the adult modal age at death. The mode is also called normal length of life (e_N or M) and represents the most often recorded age of deaths of individuals. In life tables it is identified as the age when the maximum of d_x is reached, i.e. $\text{argmax}_x\, d_x$.

The objective of the article is to introduce and compare several methods of smoothing mortality curve, estimating mode and describe statistically age distribution of deaths in the population, separately for males and females. Among considered methods there are statistical models (m_x is specific mortality rate, q_x is probability of dying, μ_x is intensity of mortality for the age x):

- Gompertz–Makeham model: $\mu_x = a + b.c^x$, a, b and c are parameters of the model for $x \in \langle 60;\ 82\rangle$ and : $\mu_x = a + b.c^{x_0 + \frac{1}{\gamma}.\ln[\gamma(x - x_0) + 1]}$, a, b, c and γ are parameters of the model for $x \in \langle 83;\ 110\rangle$
- logistic models:
  - Himes-Preston-Condran model: $m_x = \dfrac{b.e^{a.x}}{1 + b.e^{a.x}}$, where a and b are parameters of the model,
  - Heligman-Pollard model: $q_x = \dfrac{b.e^{a.x}}{1 + b.e^{a.x}}$, where a and b are parameters of the model,
  - Thatcher model: $\mu_x = \dfrac{z}{1 + z} + \gamma$, where $z = \alpha.e^{\beta.x}$, α, β and γ are parameters of the model,
  - Kannisto model: $\mu_x = \dfrac{e^{[\Theta_0 + \Theta_1(x - 80)]}}{1 + e^{[\Theta_0 + \Theta_1(x - 80)]}}$, Θ_0 and Θ_1 are nonnegative parameters of the model,
- cubic model: $\mu_x = B.C^x.D^{x^2}.E^{x^3}$, where B, C, D and E are parameters of the model,
- polynomial model: $m_x = a + b.x + c.x^2$ or $m_x = a + b.x + c.x^2 + d.x^3$, where a, b, c and d are parameters of the model,
- Coale-Kisker model: $m_x = e^{a.x^2 + b.x + c}$, where a, b and c are parameters of the model,
- Denuit-Goderniaux model: $\ln \hat{q}_x = a + b.x + c.x^2 + \varepsilon_x$, where a, b and c are parameters of the model, ε_x is an error term,
- Weibull model: $m_x = b.x^a$, where a and b are parameters of the model,
  - P-spline-smoothing method based on penalised Poisson likelihood: $\mu_x = e^{B(x).\hat{\alpha}}$, where $\ln(E(D)) = \ln(E.\mu) = \ln E + \ln \mu$, where D, E and μ are observed death, person-years lived and intensity of mortality vectors, each one including all the age-specific information; parameters of the model; this regression model is estimated with a flexible nonparametric approach based on P-splines, which combines concept of B-splines and concept of penalised likelihood approach.
- asymmetry models: based on the additional term $I_{\{x \geq K\}}$, which is a dichotomous variable or term $I_{\{x \geq K\}}.\mu_x$ or other combination.

Second, the trend in the period 1950–2014 is analyzed for the case of the Czech Republic. Since the Czech Republic went through different phases regarding mortality and fertility trends with significant changes after 1990, the history can be split into several intervals. For these periods it is analyzed how parameters of parametric distributions change over a time and it is discussed further, whether this approach can be used for well-founded and justified projection to the future.

Finally, the focus is concentrated on the asymmetry of the age distribution of deaths. Based on the estimated age distribution moments can be expressed, besides the first moment (expected value) also the second moment (variability), the third moment (skewness) and the fourth moment (kurtosis). This is the foundation for the estimate of maximal age that is reached by an individual in the population (denoted usually as ω–1) and, especially, how this maximum is evolving over a time and for each gender, i.e. separately for males and for females. More, estimate of age distribution of deaths and its characteristics leads to the assessment of

i) measurement of longevity,
ii) estimation of modal age at death for males and females,
iii) modelling trend in modal age evolution,
iv) evaluation of males and females differences, including males–females convergence,
v) estimation of maximal age for males and females,
vi) estimation of maximal age evolution,
vii) comparison of various methods of mortality smoothing, modal age estimates among each other,
viii) comparison of estimates of modal age (result vii) with other measures, such as life expectancy (life expectancy at birth e^0_0, life expectancy at other age, for example 65+e_{65}), median age and life expectancy in health.

Acknowledgement

This article was supported by the Grant Agency of the Czech Republic No. GA ČR 15-13283S under the title *Projection of the Czech Republic Population According to Educational Level and Marital Status* and was processed with contribution of long term institutional support of research activities by Faculty of Informatics and Statistics, University of Economics in Prague.

Current address
Martina Miskolczi, Ing. Mgr. MBA, Ph.D.
University of Economics in Prague
Faculty of Informatics and Statistics
Department of Demography
Nam. W. Churchilla 4
130 67 Prague 3
Czech Republic
+420 602 609 830, martina.miskolczi@vse.cz

Jitka Langhamrová, doc. Ing. CSc.
University of Economics in Prague
Faculty of Informatics and Statistics
Department of Demography

Nam. W. Churchilla 4
130 67 Prague 3
Czech Republic
+420 224 095 247, langhamj@vse.cz

Jana Langhamrová, Ing.
University of Economics in Prague
Faculty of Informatics and Statistics
Department of Statistics and Probability
Nam. W. Churchilla 4
130 67 Prague 3
Czech Republic
+420 224 095 247, jana.langhamrova@vse.cz